

# DeltaTree: A Locality-aware Concurrent Search Tree

Ibrahim Umar  
Department of Computer  
Science  
University of Tromsø, Norway  
[ibrahim.umar@uit.no](mailto:ibrahim.umar@uit.no)

Otto J. Anshus  
Department of Computer  
Science  
University of Tromsø, Norway  
[otto@cs.uit.no](mailto:otto@cs.uit.no)

Phuong Hoai Ha  
Department of Computer  
Science  
University of Tromsø, Norway  
[phuong@cs.uit.no](mailto:phuong@cs.uit.no)

## ABSTRACT

Like other fundamental abstractions for high-performance computing, search trees need to support both high concurrency and data locality. However, existing locality-aware search trees based on the van Emde Boas layout (vEB-based trees), poorly support *concurrent* (update) operations.

We present DeltaTree, a practical locality-aware concurrent search tree that integrates both locality-optimization techniques from vEB-based trees, and concurrency-optimization techniques from highly-concurrent search trees. As a result, DeltaTree minimizes data transfer from memory to CPU and supports high concurrency. Our experimental evaluation shows that DeltaTree is up to 50% faster than highly concurrent B-trees on a commodity Intel high performance computing (HPC) platform and up to 65% faster on a commodity ARM embedded platform.

## Categories and Subject Descriptors

D.1.3 [Concurrent programming]: Parallel programming; D.4.8 [Performance]: Measurements—*performance evaluation, concurrent performance*; G.1.0 [General]: Parallel algorithms—*complexity measures, performance measures*

## Keywords

Performance evaluation; concurrent algorithms; data locality; multi-core processors; memory systems

## 1. INTRODUCTION

The conventional van Emde Boas (vEB) layout based trees are examples of locality-aware search trees found in several research on cache-oblivious (CO) data structure [1–5].

The main feature of the vEB layout is that the cost of any search is  $O(\log_B N)$  memory transfers, where  $N$  is the tree size and  $B$  is the *unknown* memory block size in the CO model [5]. As such, its search is cache-oblivious. The search cost is optimal and matches that of B-trees. However, B-trees

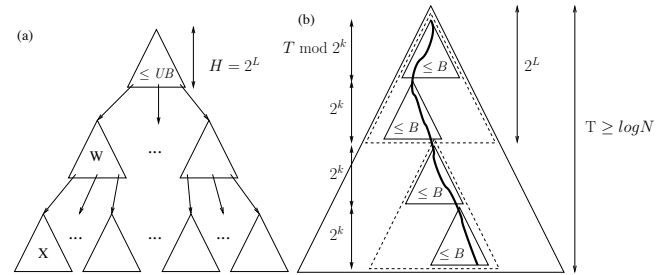


Figure 1: (a) New concurrency-aware vEB layout. (b) Search using concurrency-aware vEB layout.

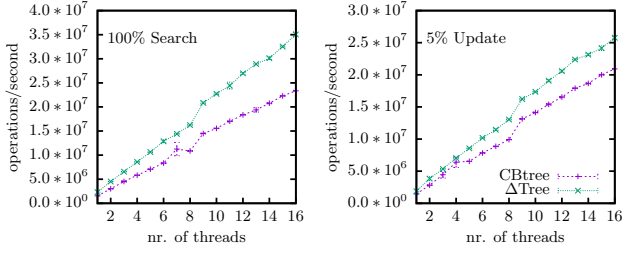
requires the memory block size  $B$  to be *known in advance*. To accomplish optimal search cost, at any level of detail each subtree in the vEB layout is stored in a contiguous block of memory.

Although the conventional vEB layout benefits from data locality, it poorly supports *concurrent* update operations. Inserting (or deleting) a node at position  $i$  in the contiguous block of memory storing the tree may trigger a restructure of a large part of the tree. Even worse, we will need to allocate a new contiguous block of memory for the whole tree if the previously allocated block of memory for the tree runs out of space [4]. Note that we cannot use dynamic node allocation via pointers since at *any* level of detail, each subtree in the vEB layout must be stored in a *contiguous* block of memory.

## 2. RELAXED CO MODEL AND CONCURRENCY-AWARE VEB LAYOUT

In order to make the vEB layout suitable for highly concurrent data structures with concurrent update operations, we introduce a novel *concurrency-aware* dynamic vEB layout. Our key idea is that if we know an *upper bound*  $UB$  on the unknown memory block size  $B$ , we can support dynamic node allocation via pointers while maintaining the optimal search cost of  $O(\log_B N)$  memory transfers without knowing  $B$  (cf. Lemma 2.1). This idea is based on that in practice it is not feasible to keep the vEB layout in a contiguous block of physical memory greater than some upper bound set by the underlying system physical page size (frame size) and cache-line size.

Figure 1a illustrates the new concurrency-aware vEB layout based on the relaxed CO model. The memory transfer cost for search operations in the new concurrency-aware vEB layout is the same as that of the conventional vEB layout (cf.



**Figure 2: Performance comparison using  $2^{22}$  initial values on an Intel HPC platform with dual Intel Xeon E5-2670 CPUs.**

Lemma 2.1). However, the concurrency-aware vEB supports high concurrency for update operations.

We define *relaxed cache-oblivious* algorithms as cache-oblivious (CO) algorithms with the restriction that an upper bound  $UB$  on the unknown memory block size  $B$  is known in advance. As long as an upper bound on all the block sizes of multilevel memory is known, the new relaxed CO model maintains the key features of the original CO model [5]. This enables algorithm designs that can utilize fine-grained data locality in the multilevel memory hierarchy of modern architectures.

**LEMMA 2.1.** *For any upper bound  $UB$  on the unknown memory block size  $B$ , a search in a complete binary tree with the concurrency-aware vEB layout achieves the optimal memory transfer  $O(\log_B N)$ , where  $N$  and  $B$  are the tree size and the unknown memory block size in the CO model, respectively.*

**PROOF.** (Sketch) Figure 1b illustrates the proof. Let  $k$  be the coarsest level of detail such that every recursive subtree contains at most  $B$  nodes. Since  $B \leq UB$ ,  $k \leq L$ , where  $L$  is the coarsest level of detail at which every recursive subtree ( $\Delta$ Nodes) contains at most  $UB$  nodes. Consequently, there are at most  $2^{L-k}$  subtrees along the search path in a  $\Delta$ Node and no subtree of depth  $2^k$  is split due to the boundary of  $\Delta$ Nodes. Namely, triangles of height  $2^k$  fit within a dashed triangle of height  $2^L$  in Figure 1b.

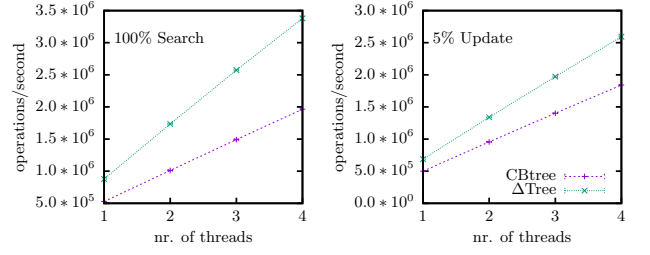
Because at any level of detail  $i \leq L$  in the concurrency-aware vEB layout, a recursive subtree of depth  $2^i$  is stored in a contiguous block of memory, each subtree of depth  $2^k$  within a  $\Delta$ Node is stored in at most two memory blocks of size  $B$  (depending on the starting location of the subtree in memory). Since every subtree of depth  $2^k$  fits in a  $\Delta$ Node (i.e., no subtree is stored across two  $\Delta$ Nodes), every subtree of depth  $2^k$  is stored in at most two memory blocks of size  $B$ .

Since the tree has height  $T$ ,  $\lceil T/2^k \rceil$  subtrees of depth  $2^k$  are traversed in a search and thereby at most  $2\lceil T/2^k \rceil$  memory blocks are transferred.

Since a subtree of height  $2^{k+1}$  contains more than  $B$  nodes,  $2^{k+1} \geq \log_2(B+1)$ , or  $2^k \geq \frac{1}{2} \log_2(B+1)$ .

We have  $2^{T-1} \leq N \leq 2^T$  since the tree is a *complete* binary tree. This implies  $\log_2 N \leq T \leq \log_2 N + 1$ .

Therefore, the number of memory blocks transferred in a search is  $2\lceil T/2^k \rceil \leq 4\lceil \frac{\log_2 N + 1}{\log_2(B+1)} \rceil = 4\lceil \log_{B+1} N + \log_{B+1} 2 \rceil = O(\log_B N)$ , where  $N \geq 2$ .  $\square$



**Figure 3: Performance comparison using  $2^{21}$  initial values on an ARM platform with a Samsung Exynos 5410 octa-core ARM CPU.**

### 3. CONTRIBUTIONS

We devised a new *relaxed* cache-oblivious model and a novel *concurrency-aware* vEB layout that makes the vEB layout suitable for highly-concurrent data structures even with update operations (cf. Figure 1). The concurrency-aware vEB layout supports dynamic node allocation via pointers while maintaining the optimal search cost of  $O(\log_B N)$  memory transfers without knowing the exact value of block size  $B$  (cf. Lemma 2.1).

Based on the new concurrency-aware vEB layout, we developed a new locality-aware concurrent search tree called DeltaTree ( $\Delta$ Tree).

We experimentally evaluated<sup>1</sup>  $\Delta$ Tree on a commodity Intel HPC platform (cf. Figure 2) and an ARM embedded platform (cf. Figure 3).  $\Delta$ Tree is up to 50% and 65% faster than CBtree [6], a highly-concurrent B-tree, on Intel HPC and ARM platforms, respectively.

### 4. ACKNOWLEDGMENTS

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<sup>1</sup>Using 5 million operations in two micro-benchmarks: 100% search and 5% update (95% search) using random numbers.